Mass Differences within Isotopic Multiplets in a SUSY Electro-weak Theory

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Abstract

Based on the idea that electromagnetism is responsible for the mass differences within isotopic multiplets, and possibly also for the whole mass of the electron, a supersymmetric gauge theoretical model based on the group $SU(2)_L \times SU(2)_R \times U(1)_Y$ is constructed. Under some reasonable assumptions to the SUSY particle spectrum, a correct sign for the mass difference within an isotopic multiplet is obtained. This might provide a possible scenario to understand the old puzzle of the proton-neutron mass difference.

Introduction− It had challenged and frustrated generations of physicists to apply the idea that electromagnetic and weak interactions are responsible for mass differences within isotopic multiplets, e.g. $\Delta m|_{d-u}$, and possibly also for the whole mass of the electron to calculations for they always gave a wrong sign[1,2]. In the last few decades, as physicists understand more the interactions and fundamental structures of matters, they tend to believe that[1] isotopic symmetry is not a fundamental symmetry in strong interaction, and the false impression is due to the small u-d quark mass difference, though it is comparable to the quark masses themselves (about a few Mev), on the typical strong-interaction scale (about 200-400 Mev). This sort of view may well be correct, but a rigorous experimental proof will not be easy[1]. For many reasons, the idea that $\Delta m|_{d-u}$ is due to electromagnetic and weak interactions is still very attractive, although there are difficulties in calculations of some physical quantities. Alternatively, it is natural to ask ourselves whether the previous incorrect results in the calculations are due to the limit of our theoretical understanding of the nature? For this reason, the author and his collaborator once considered a supersymmetric extension of an $SU(2) \times U(1)$ toy model[3] and nicely obtained a correct sign for the mass difference within an isodoublet. Although it is just a toy model, the result is still very encouraging. The question is whether we can construct a more realistic model which should be consistent with the Standard Model.

In this letter, I will study a supersymmetric extension of an $SU(2)_L \times SU(2)_R \times U(1)_Y$ model, which was originally suggested by S. Weinberg[2], and will discuss the mass difference within an isodoublet in this model. It is supposed that the weak and electromagnetic gauge group $SU(2)_L \times U(1)_Y$ is part of a larger gauge group $SU(2)_L \times SU(2)_R \times U(1)_Y$. We don't see effects of the gauge bosons associated with such transformations, so we must suppose that they are very heavy[1]. Fortunately these vector bosons can be almost arbitrarily heavy, and still produce the necessary mass shifts. The mass difference within an isotopic multiplet in this model is due to the "type 1" mass relation[2], which guarantees

that the mass difference does not arise from graphs involving virtual scalar bosons, whose properties are almost entirely unknown.

The Supersymmetric $SU(2)_L\times SU(2)_R\times U(1)_Y$ *Model*— The supersymmetric generalization[4] of this model consists of the fields listed in Table 1. Comparing to the original gauge field model[2], here Φ_2 is added to take care of the problem in β decay (see Freedman's paper in [2]), and Φ_1' and Φ_2' are needed for generating masses for the supersymmetric partners of the gauge particles, and N_i's are responsible for the existence of a unique ground state which breaks $SU(2)_L \times SU(2)_R \times U(1)_Y$ to $U(1)_{EM}$ at tree level for this unbroken supersymmetric model.

The scalar potential V in the SUSY $SU(2)_L \times SU(2)_R \times U(1)_Y$ model is

$$V = \frac{1}{2}[D^a_L D^a_L + D^a_R D^a_R + (D')^2] + F^*_i F_i, \tag{1}$$

where

$$D^a_{L(R)} = \frac{g_{L(R)}}{2} A^*_{iL(R)} \tau^a_{ij} A_{jL(R)}, \quad D' = \frac{g_Y y_i}{2} A^*_i A_i, \quad F_i = \frac{\partial W}{\partial A_i}, \tag{2}$$

with $A_{iL(R)}$ as scalar fields transforming as doublets in $SU(2)_{L(R)}$ respectively, A_i as scalar fields listed in Table 1, and $W = (h_m \epsilon_{ij} \Phi^i_1 \Phi'^j_1 + l_m \epsilon_{ij} \Phi^i_2 \Phi'^j_2 + k_m H_{ij} H_{ji} + s_m) N_m + f \epsilon_{ij} F^i_R H^{jk} F^k_L$, where $m = 1, 2, 3$.

From eqs.1 and 2, the nonzero vacuum expectation values of the scalar fields are

$$\begin{aligned} H &= \frac{1}{\sqrt{2}}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \quad \Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_1' = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \\ \Phi_2 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \Phi_2' = \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \end{aligned} \tag{3}$$

which break $SU(2)_L \times SU(2)_R \times U(1)_Y$ down to $U(1)_{EM}$. The constants v, v_1 and v_2 are related to h_m, l_m, k_m, s_m of eq.2 by $\frac{1}{2}h_m v_1^2 + \frac{1}{2} l_m v_2^2 + k_m v^2 + s_m = 0$. As can be seen from eqs.1, 2 and 3, the scalar potential has $V_{min} = 0$, thus implying that the theory remains supersymmetric.

By considering the interaction terms after spontaneous gauge symmetry breaking, we

can have the following mass eigenstates,

$$\begin{pmatrix} A^\mu(\lambda_A) \\ Z_1^\mu(\lambda_{Z_1}) \\ Z_2^\mu(\lambda_{Z_2}) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} A_3^{\mu L}(\lambda_L^3) \\ A_3^{\mu R}(\lambda_R^3) \\ B^\mu(\lambda_Y) \end{pmatrix},$$

$$\zeta_A = \zeta_1 = \begin{pmatrix} -i\lambda_A \\ i\bar{\lambda}_A \end{pmatrix}, \quad \zeta_{Z_{1,2}} = \zeta_{2,3} = \begin{pmatrix} -i\lambda_{2,3} \\ i\bar{\lambda}_{Z_{1,2}} \end{pmatrix}, \tag{4}$$

with

$$\begin{aligned} -i\lambda_i &= \frac{1}{2m_{Z_i}}[v_1(g_L a_{i1} - g_Y a_{i3})(\tilde{\Phi}_1'^1 - \tilde{\Phi}_1^2) + v(g_L a_{i1} - g_R a_{i2})(\tilde{H}_{11} - \tilde{H}_{22}) + \\ & \quad v_2(g_L a_{i2} - g_Y a_{i3})(\tilde{\Phi}_2'^1 - \tilde{\Phi}_2^2)], \\ i &= 2,3, \quad m_A = m_{\zeta_A} = 0, \\ m^2{}_{Z_{1,2}} &= m^2{}_{\zeta_{Z_{1,2}}} = \{[g_L^2(v^2+v_1^2) + g_R^2(v^2+v_2^2) + g_Y^2(v_1^2+v_2^2)] \pm ([g_L^2(v^2+v_1^2) + \\ & \qquad g_R^2(v^2+v_2^2) + g_Y^2(v_1^2+v_2^2)]^2 - 4(v^2v_1^2 + v^2v_2^2 + v_1^2v_2^2)(g_L^2 g_R^2 + \\ & \qquad (g_L^2 g_R^2 + g_L^2 g_Y^2 + g_R^2 g_Y^2))^2\}/8, \end{aligned} \tag{5}$$

where the (a_{ij}) is orthogonality matrices. Their elements can be determined by orthogonality condition and the lagrangian.[5]

The Lagrangian of the SUSY model can be written as

$$\begin{aligned} \mathcal{L}_{int} &= \bar{\psi} i\gamma^\mu (g_L \vec{\tau}\cdot\vec{A}_\mu^{\ L} P_L + g_R \vec{\tau}\cdot\vec{A}_\mu^{\ R} P_R + g_Y y_L B_\mu)\psi/2 + g_L \bar{\tilde{\omega}}_L^- P_L \psi_2 \tilde{\psi}_{1L}^* + \\ & \quad g_L \bar{\psi}_1 P_R \tilde{\omega}_L^+ \tilde{\psi}_{2L} + g_L \bar{\tilde{\omega}}_L^+ P_L \psi_1 \tilde{\psi}_{2L}^* + g_L \bar{\psi}_2 P_R \tilde{\omega}_L^- \tilde{\psi}_{1L} - g_R \bar{\psi}_1 P_L \tilde{\omega}_R^{-c} \tilde{\psi}_{2R} - \\ & \quad g_R \bar{\tilde{\omega}}_R^{+c} P_R \psi_2 \tilde{\psi}_{1R}^* - g_R \bar{\psi}_2 P_L \tilde{\omega}_R^{+c} \tilde{\psi}_{1R} - g_R \bar{\tilde{\omega}}_R^{-c} P_R \psi_1 \tilde{\psi}_{2R}^* + \\ & \quad \{(g_L a_{j1} + g_Y y_L a_{j3})[\bar{\zeta}_j P_L \psi_1 \tilde{\psi}_{1L}^* + \bar{\psi}_1 P_R \zeta_j \tilde{\psi}_{1L}] - \\ & \quad (g_L a_{j1} - g_Y y_L a_{j3})[\bar{\zeta}_j P_L \psi_2 \tilde{\psi}_{2L}^* + \bar{\psi}_2 P_R \zeta_j \tilde{\psi}_{2L}] + \\ & \quad (g_R a_{j2} - g_Y y_L a_{j3})[\bar{\psi}_2 P_L \zeta_j^c \tilde{\psi}_{2R} + \bar{\zeta}_j^c P_R \psi_2 \tilde{\psi}_{2R}^*] - \\ & \quad (g_R a_{j2} + g_Y y_L a_{j3})[\bar{\psi}_1 P_L \zeta_j^c \tilde{\psi}_{1R} + \bar{\zeta}_j^c P_R \psi_1 \tilde{\psi}_{1R}^*]\}/\sqrt{2}, \end{aligned} \tag{6}$$

where $\tilde{\omega}_L^-$, $\tilde{\omega}_L^+$, $\tilde{\omega}_R^-$, $\tilde{\omega}_R^+$ are the SUSY partners of the gauge fields A_L^-, A_L^+, A_R^-, A_R^+, respectively.

Working in U gauge, from eq.6, we can obtain the second order $\Delta m|_{d-u}$ in the supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_Y$ model as

$$\Delta m|_{d-u} = -\frac{\alpha m}{2\pi}\int_0^1 dx[d_1 ln(1+\frac{1-x}{x^2}\frac{m_{Z_1}^2}{m^2}) + d_2 ln(1+\frac{1-x}{x^2}\frac{m_{Z_2}^2}{m^2})], \tag{7}$$

where m is the zeroth-order mass of the isodoublet appearing in the Lagrangian, m_{Z_1}, m_{Z_2} are given by eq.5 and $\alpha = e^2/4\pi = 1/137.04$ with e^2, d_1, d_2 defined by

$$\begin{aligned} e &= \frac{g_L g_R g_Y}{({g_L}^2{g_R}^2 + {g_L}^2{g_Y}^2 + {g_R}^2{g_Y}^2)^{1/2}}, \\ d_1 &= -\frac{a_{23}(g_L a_{21} + g_R a_{22})}{a_{13}(g_L a_{11} + g_R a_{12})}, \quad d_2 = -\frac{a_{33}(g_L a_{31} + g_R a_{32})}{a_{13}(g_L a_{11} + g_R a_{12})}, \end{aligned} \tag{8}$$

where e^2 appears in eq.7 as the coefficient of the photon term. In view of the orthogonality conditions for a_{ij}, $d_{1,2}$ satisfy $d_1 + d_2 = 1$.

Comparing to the result from the pure gauge field model[2],

$$\Delta m_{G.F.}|_{d-u} = -\frac{\alpha m}{2\pi}\int_0^1 dx(1+x)[d_1 ln(1+\frac{1-x}{x^2}\frac{m_{Z_1}^2}{m^2}) + d_2 ln(1+\frac{1-x}{x^2}\frac{m_{Z_2}^2}{m^2})], \tag{9}$$

we see that $\Delta m|_{d-u}$ is still negative although it is less negative than the result obtaining from the corresponding pure gauge field model. But the encouraging thing is that the contribution to $\Delta m|_{d-u}$ from the SUSY partners could be positive. This raises some hope for getting a right sign for $\Delta m|_{d-u}$.

As we know, if supersymmetry is really a theory describing the nature, it should be broken for no supersymmetry exhibiting in the low energy particle spectrum. Therefore, a calculation of $\Delta m|_{d-u}$ from the supersymmetric Lagrangian with spontaneous gauge symmetry breaking is not complete. We should also consider the contribution to $\Delta m|_{d-u}$ due to supersymmetry breaking.

Supersymmetry Breaking in the $SU(2)_L \times SU(2)_R \times U(1)_Y$ Model$-$ I will consider explicit soft-supersymmetry breaking in this section. When SUSY is softly broken, the possible mass terms in two component notations are,

$$\begin{aligned} &\tilde{\psi}_{iL}^* L_i^2 \tilde{m}^2 \tilde{\psi}_{iL} + \tilde{\psi}_{iR}^* R_i^2 \tilde{m}^2 \tilde{\psi}_{iR} + 2A_i \tilde{m} m Re\tilde{\psi}_{iL}^* \tilde{\psi}_{iR} - \mu_1 \epsilon^{\alpha\beta}\tilde{\Phi}_1^\alpha \tilde{\Phi}_1'^\beta - \mu_2 \tilde{H}_{ij}\tilde{H}_{ji} - \\ &\mu_3 \epsilon^{\alpha\beta}\tilde{\Phi}_2^\alpha \tilde{\Phi}_2'^\beta + (M_1/2)\lambda_L^a \lambda_L^a + (M_2/2)\lambda_R^a \lambda_R^a + (M_3/2)\lambda'\lambda'. \end{aligned} \tag{10}$$

This will lead to the mixings among different particles. In principle, it is better to obtain mass eigenstates and their corresponding masses numerically. For simplicity and analyticity, an analytically worked example is the case where $\mu_1 = \mu_2 = \mu_3 = 0$. If I further set $v_1' = v_1'' = v_1$, $v_2' = v_2'' = v_2$, $M_1 = M_2 = M_3 = M_0$, then the mass eigenstates and their masses in the SUSY breaking model will be given in the following. Note that the superpotential in this case is not necessary to have the form specified in eq.1 and it will allow a more general set of vacuum expectation values than that of eq.3, e.g.

$$\Phi_1 = \frac{v_1'}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \Phi_1' = \frac{v_1''}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \Phi_2 = \frac{v_2'}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \Phi_2' = \frac{v_2''}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{11}$$

1. Mixing of scalar-quarks:

   The mass eigenstates and their masses are

$$\begin{aligned} \tilde{\psi}_{iI} &= \tilde{\psi}_{iL} cos\theta_i + \tilde{\psi}_{iR} sin\theta_i, \quad \tilde{\psi}_{iII} = -\tilde{\psi}_{iL} sin\theta_i + \tilde{\psi}_{iR} cos\theta_i, \\ tan2\theta_i &= 2A_i m/[(L_i^2 - R_i^2)\tilde{m}^2], \\ \mu^2_{iI,II} &= m^2 + \{(L_i^2 + R_i^2)\tilde{m}^2 \pm [(L_i^2 - R_i^2)^2\tilde{m}^4 + 4{A_i}^2 m^2 \tilde{m}^2]^{1/2}\}/2, \quad i = 1, 2. \end{aligned} \tag{12}$$

2. Mixing of charged gauginos and higginos:

   Defining

$$\begin{aligned} \Psi_j^+ &= (-i\lambda_L^+,\ (v_1\tilde{\Phi}_1^1 + v\tilde{H}_{12})/(v^2 + v_1^2)^{1/2},\ -i\lambda_R^+,\ (v_2\tilde{\Phi}_2^1 - v\tilde{H}_{12})/(v^2 + v_2^2)^{1/2}), \\ \Psi_j^- &= (-i\lambda_L^-,\ (v_1\tilde{\Phi}_1'^2 + v\tilde{H}_{12})/(v^2 + v_1^2)^{1/2},\ -i\lambda_R^-,\ (v_2\tilde{\Phi}_2'^2 - v\tilde{H}_{21})/(v^2 + v_2^2)^{1/2}), \\ j &= 1, 2, 3, 4, \end{aligned} \tag{13}$$

the mass eigenstates and their masses are given by

$$\tilde{\chi}_i = \begin{pmatrix} \chi_i^+ \\ \bar{\chi}_i^- \end{pmatrix}, \qquad \chi_i^+ = V_{ij}\Psi_j^+, \quad \chi_i^- = U_{ij}\Psi_j^-;$$

$$\begin{aligned} \tilde{M}_{1,2} &= \{[M_0^2 + 2g_L^2(v^2 + v_1^2)]^{1/2} \pm M_0\}/2, \\ \tilde{M}_{3,4} &= \{[M_0^2 + 2g_R^2(v^2 + v_2^2)]^{1/2} \pm M_0\}/2, \end{aligned} \tag{14}$$

where the unitary matrices U, V are given by

$$\begin{aligned} U &= \begin{pmatrix} O_1 & 0 \\ 0 & O_2 \end{pmatrix}, \quad V = \begin{pmatrix} \sigma_3 O_1 & 0 \\ 0 & \sigma_3 O_2 \end{pmatrix}, \quad \text{with} \quad O_i = \begin{pmatrix} cos\phi_i & sin\phi_i \\ -sin\phi_i & cos\phi_i \end{pmatrix}, \\ i &= 1, 2, \quad cos\phi_1' = [\tilde{M}_1/(\tilde{M}_1 + \tilde{M}_2)]^{1/2}, \quad cos\phi_2' = [\tilde{M}_3/(\tilde{M}_3 + \tilde{M}_4)]^{1/2}. \end{aligned} \tag{15}$$

3. Mixing of neutral gauginos and higginos:

Defining

$$\Psi_j^0 \quad = (-i\lambda_A,\ -i\lambda_{Z_1},\ -i\lambda_2,\ -i\lambda_{Z_2},\ -i\lambda_3), \tag{16}$$

where $-i\lambda_{Z_{2,3}}$ are given in eq.5, then the mass eigenstates and their masses are given by

$$\tilde{\chi}_i^0 = \left(\begin{array}{c} \chi_i^0 \\ \bar{\chi}_i^0 \end{array} \right), \quad \chi_i^0 = N_{ij}\Psi_j^0, \quad i = 1, \cdots, 5;$$

$$\begin{aligned} &\tilde{N}_1 = M_0,\ \tilde{N}_{2,3} = ({m_{Z_1}}^2 + {M_0}^2/4)^{1/2} \pm M_0/2, \\ &\tilde{N}_{4,5} = ({m_{Z_2}}^2 + {M_0}^2/4)^{1/2} \pm M_0/2, \end{aligned} \tag{17}$$

with the matrix N as

$$N \quad = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & cos\phi_1 & sin\phi_1 & 0 & 0 \\ 0 & -isin\phi_1 & icos\phi_1 & 0 & 0 \\ 0 & 0 & 0 & cos\phi_2 & sin\phi_2 \\ 0 & 0 & 0 & -isin\phi_2 & icos\phi_2 \end{pmatrix}$$

$$cos\phi_1 \quad = \quad (\frac{\tilde{N}_2}{\tilde{N}_2 + \tilde{N}_3})^{1/2}, \quad cos\phi_2 = (\frac{\tilde{N}_4}{\tilde{N}_4 + \tilde{N}_5})^{1/2}. \tag{18}$$

$Mass\ Difference\ within\ an\ Isodoublet\ in\ the\ Soft-Broken\ Supersymmetric\ SU(2)_L \times SU(2)_R \times U(1)_Y\ Model-$ The SUSY breaking will lead to a different interaction Lagrangian from the one given in eq.6, and we should also expect different mass difference within an isotopic multiplet from the one given in eq.7. Just for simplicity, instead of doing a general numerical studies, I will do an analytic study for one set of parameters to show the possibility of obtaining the right sign for the mass difference within an isodoublet.

Supposed I set $\mu_1 = \mu_2 = \mu_3 = 0$, $M_1 = M_2 = M_3 = M_0$, $v_1' = v_1'' = v_1$, $v_2' = v_2'' = v_2$ for the parameters appearing in the soft-SUSY breaking terms, then I can substitute eqs.12, 14, 17 into eq.6, and obtain the Lagrangian for the SUSY breaking $SU(2)_L \times SU(2)_R \times U(1)_Y$ model. It is not hard to realize that there are two kinds of diagrams contributing to $\Delta m|_{d-u}$ from the SUSY breaking Lagrangian.

Kind 1 is of the form $P_L\ \Gamma\ P_R$ or $P_R\ \Gamma\ P_L$, which, being similar to the integrals I got in Section II, would be proportional to the fermion mass of the isodoublet.

Kind 2 is of the form $P_L\ \Gamma\ P_L$ or $P_R\ \Gamma\ P_R$, which we did not see before. As we will see, this will play a very important role in getting a right sign for $\Delta m|_{d-u}$.

Define

$$J(m, m_1, m_2, m_3) = \frac{-im^2}{16\pi^2}\int_0^1 dz\ ln\frac{z(z-1)m^2 + zm_3{}^2 + (1-z)m_1^2}{z(z-1)m^2 + zm_3^2 + (1-z)m_2^2}. \tag{19}$$

If I use $\Delta m_{1,2}|_{d-u}$ to represent contributions from kind 1, 2 respectively, and further set $\mu_{1I} = \mu_{2I}$, $\mu_{1II} = \mu_{2II}$, which also lead to $\theta_1 = \theta_2$, detailed calculations show that

$$\begin{aligned}
\Delta m_1|_{d-u} &> -\frac{\alpha m}{2\pi}\int_0^1 dx(1+x)[d_1 ln(1+\frac{1-x}{x^2}\frac{m_{Z_1}^2}{m^2}) + d_2 ln(1+\frac{1-x}{x^2}\frac{m_{Z_2}^2}{m^2})] \\
\Delta m_2|_{d-u} &= (i\tilde{N}_m/2m^2)Re(N_{m,j+c_j}N_{m,k+c_k})sin2\theta_1 J(m,\mu_{1I},\mu_{1II},\tilde{N}_m)g_Y y_L \cdot \\
&\qquad (g_L a_{j1}a_{k3} + g_R a_{j3}a_{k2}),
\end{aligned} \tag{20}$$

where in $\Delta m_1|_{d-u}$, some terms give positive, and some negative contributions to $\Delta m|_{d-u}$, and in $\Delta m_2|_{d-u}$, $c_1 = c_2 = 0$, $c_3 = 1$; $j, k = 1, 2, 3$. Since

$$J(m,\mu_{1I},\mu_{1II},\tilde{N}_m) > J(m,\mu_{1I},\mu_{1II},\tilde{N}_n) \quad \text{for } \mu_{1I} > \mu_{1II},\ \tilde{N}_m < \tilde{N}_n, \tag{21}$$

using eqs.21, 18, 17, I can further rewrite $\Delta m_2|_{d-u}$ in eq.20 as

$$\begin{aligned}
\Delta m_2|_{d-u} &> (ig_L e^2 M_0 sin2\theta_1/m^2)\{d_1[J(m,\mu_{1I},\mu_{1II},M_0) - J(m,\mu_{1I},\mu_{1II},\tilde{N}_3)] + \\
&\quad d_2[J(m,\mu_{1I},\mu_{1II},M_0) - J(m,\mu_{1I},\mu_{1II},\tilde{N}_5)]\} \\
&= \kappa M_0 > 0,
\end{aligned} \tag{22}$$

provided we have $M_0 < \tilde{N}_3$ and $M_0 < \tilde{N}_5$, i.e. $m_{Z_1} > \sqrt{2}M_0$ and $m_{Z_2} > \sqrt{2}M_0$.

For m_{Z_1} and m_{Z_2} are at least in the order of magnitude of 10^2 Gev, it is not hard to have $M_0 >> m$ ($\sim$ a few Mev) but still satisfy the requirement $m_{Z_1} > \sqrt{2}M_0$ and $m_{Z_2} > \sqrt{2}M_0$. This condition is not inconsistent with the common expectation in supersymmetry phenomenology[4].

We notice from eq.20 that $\Delta m_1|_{d-u}$ at most increases with logarithm of m_{Z_i}/m, but detailed analysis to eqs.22, 19 shows that κ increases with μ_{1I}/μ_{1II}, i.e. κ can be a not very small value by proper choices of μ_{1I} and μ_{1II}, e.g. $\kappa \sim 0.001$[5], and also $M_0 >> m$ ($\sim$ few Mev). Therefore, I can be definite to expect that

$$\Delta m|_{d-u} = \Delta m_1|_{d-u} + \Delta m_2|_{d-u} \sim \Delta m_2|_{d-u} > 0, \tag{23}$$

for proper choices of μ_{1I} and μ_{1II} in the case of $\mu_1 = \mu_2 = \mu_3 = 0$, $M_1 = M_2 = M_3 = M_0$, $v_1' = v_1'' = v_1$, $v_2' = v_2'' = v_2$, and $\mu_{1I} = \mu_{2I}$, $\mu_{1II} = \mu_{2II}$.

Discussion$-$ In this papaer, the possibility of obtaining the right sign for the mass difference within an isotopic multiplets in a supersymmetric gauge theory is raised. In

Table 1: The fields in the SUSY $SU(2)_L \times SU(2)_R \times U(1)_Y$ model. Note: (1) The charge is obtained via $Q = T_{L3} + T_{R3} + Y/2$; (2) $\psi_i = \binom{\psi_{iL}}{\psi_{iR}}$; (3) For $\binom{p}{n}$ and $\binom{u}{d}$, we will have $y_L = -y_R = 1$, and $y_L = -y_R = 1/3$ respectively.

	Boson Fields	Fermionic Partners	$SU(2)_L$	$SU(2)_R$	Y
Gauge multiplets	$\vec{A}_\mu^L$	$\vec{\lambda}_L$	Triplet		0
	$\vec{A}_\mu^R$	$\vec{\lambda}_R$		Triplet	0
	B_μ	λ'	Singlet	Singlet	0
Matter multiplets					
Scalar fermions	$\tilde{\psi}_{jL} = (\tilde{\psi}_{1L},\ \tilde{\psi}_{2L})$	$\psi_{jL} = (\psi_{1L},\ \psi_{2L})$	Doublet	Singlet	y_L
	$\tilde{\psi}_{jR} = (-\tilde{\psi}_{2R}^*,\ \tilde{\psi}_{1R}^*)$	$\psi_{jR} = (-\psi_{2R},\ \psi_{1R})$	Singlet	Doublet	y_R
Higgs Bosons	$\Phi_1 = (\Phi_1^1,\ \Phi_1^2)$	$\tilde{\Phi}_1 = (\tilde{\Phi}_1^1,\ \tilde{\Phi}_1^2)$	Doublet	Singlet	1
	$\Phi_1' = (\Phi_1'^1,\ \Phi_1'^2)$	$\tilde{\Phi}_1' = (\tilde{\Phi}_1'^1,\ \tilde{\Phi}_1'^2)$	Doublet	Singlet	-1
	$\Phi_2 = (\Phi_2^1,\ \Phi_2^2)$	$\tilde{\Phi}_2 = (\tilde{\Phi}_2^1,\ \tilde{\Phi}_2^2)$	Singlet	Doublet	1
	$\Phi_2' = (\Phi_2'^1,\ \Phi_2'^2)$	$\tilde{\Phi}_2' = (\tilde{\Phi}_2'^1,\ \tilde{\Phi}_2'^2)$	Singlet	Doublet	-1
	$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$	$\tilde{H} = \begin{pmatrix} \tilde{h}_{11} & \tilde{h}_{12} \\ \tilde{h}_{21} & \tilde{h}_{22} \end{pmatrix}$	Doublet	Doublet	0
	$N_1,\ N_2,\ N_3$	$\tilde{N}_1,\ \tilde{N}_2,\ \tilde{N}_3$	Singlet	Singlet	0

the scheme I used, SUSY breaking, which is what we expected if supersymmetry is really a symmetry in the real world, play a very important role. The model is not inconsistent with the Standard Model and the present experimental limits[4]. Certainly, I only choose a special set of the parameters for calculation simplification. A thorough numerical study for the parameters would be nice for obtaining the particle spectrums. If the mechanism I used really say something about the nature, we may be able to use $\Delta m|_{d-u}$ as a constraint to the parameters appearing in soft-supersymmetry breaking. Of course, I only raise a possibility here. Even under the current framework, much more works are still needed, especially considering the ideas developed in the last few decades in field theories and particle physics.

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